Correlation with the 2019 Updated AP® Calculus AB and BC Course and Exam Description

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Calculus for AP®

SECOND EDITION

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Course: AP® Calculus AB and BC

Unit 1: Limits and Continuity

Suggested Length: AB ~22-23 class periods
BC ~13-14 class periods

AP Exam Weighting: AB 10-12%
BC 4-7%

Big Ideas: Change (CHA); Limits (LIM); Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 1.1: Introducing Calculus: Can Change Occur at an Instant?	**CHA-1:** Calculus allows us to generalize knowledge about motion to diverse problems involving change. **CHA-1.A:** Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.	**CHA-1.A.1:** Calculus uses limits to understand and model dynamic change.	Section 1.1, pp. 58-59
		CHA-1.A.2: Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.	Section P.2, p. 16 Section 1.1, pp. 58-59
		CHA-1.A.3: The limit concept allows us to define instantaneous rate of change in terms of average rates of change.	Section 1.1, pp. 58-59
Topic 1.2: Defining Limits and Using Limit Notation	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.A:** Represent limits analytically using correct notation.	**LIM-1.A.1:** Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.	Section 1.2, p. 65

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	LIM-1: Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.B:** Interpret limits expressed in analytic notation.	**LIM-1.B.1:** A limit can be expressed in multiple ways, including graphically, numerically, and analytically.	Section 1.2, pp. 65-71 Section 1.3, pp. 76-83
Topic 1.3: Estimating Limit Values from Graphs	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.C:** Estimate limits of functions.	**LIM-1.C.1:** The concept of a limit includes one sided limits.	Section 1.4, pp. 89-91
		LIM-1.C.2: Graphical information about a function can be used to estimate limits.	Section 1.2, p. 66
		LIM-1.C.3: Because of issues of scale, graphical representations of functions may miss important function behavior.	Section 1.2, p. 68
		LIM-1.C.4: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.	Section 1.2, pp. 67-68
Topic 1.4: Estimating Limit Values from Tables	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.C:** Estimate limits of functions.	**LIM-1.C.5:** Numerical information can be used to estimate limits.	Section 1.2, p. 66

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 1.5: Determining Limits Using Algebraic Properties of Limits	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.D:** Determine the limits of functions using limit theorems.	**LIM-1.D.1:** One-sided limits can be determined analytically or graphically.	Section 1.4, pp. 89-91
		LIM-1.D.2: Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.	Section 1.3, pp. 76-78
Topic 1.6: Determining Limits Using Algebraic Manipulation	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.E:** Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	**LIM-1.E.1:** It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.	Section 1.3, pp. 79-83
Topic 1.7: Selecting Procedures for Determining Limits			Section 1.3, p. 79
Topic 1.8: Determining Limits Using the Squeeze Theorem	**LIM-1:** Reasoning with definitions, theorems, and properties can be used to justify claims about limits. **LIM-1.E:** Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	**LIM-1.E.2:** The limit of a function may be found by using the squeeze theorem.	Section 1.3, pp. 82-83
Topic 1.9: Connecting Multiple Representations of Limits			Section 1.2, p. 66, Example 1 Section 1.3, p. 83, Example 9 Section 1.5, p. 101, Example 1

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
			Section 1.6, p. 110, Example 2
Topic 1.10: Exploring Types of Discontinuities	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity. **LIM-2.A:** Justify conclusions about continuity at a point using the definition.	**LIM-2.A.1:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.	Section 1.4, p. 88 Section 1.5, pp. 102-104
Topic 1.11: Defining Continuity at a Point	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity. **LIM-2.A:** Justify conclusions about continuity at a point using the definition.	**LIM-2.A.2:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.	Section 1.4, pp. 87-88
Topic 1.12: Confirming Continuity over an Interval	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity. **LIM-2.B:** Determine intervals over which a function is continuous.	**LIM-2.B.1:** A function is continuous on an interval if the function is continuous at each point in the interval.	Section 1.4, pp. 87-95
		LIM-2.B.2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.	Section 1.4, p. 92
Topic 1.13: Removing Discontinuities	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.	**LIM-2.C.1:** If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value	Section 1.4, p. 88

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	LIM-2.C: Determine values of *x* or solve for parameters that make discontinuous functions continuous, if possible.	of the limit of the function as *x* approaches that point.	
		LIM-2.C.2: In order for a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.	Section 1.4, p. 88
Topic 1.14: Connecting Infinite Limits and Vertical Asymptotes	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity. **LIM-2.D:** Interpret the behavior of functions using limits involving infinity.	**LIM-2.D.1:** The concept of a limit can be extended to include infinite limits.	Section 1.5, pp. 100-104
		LIM-2.D.2: Asymptotic and unbounded behavior of functions can be described and explained using limits.	Section 1.5, pp. 100-104
Topic 1.15: Connecting Limits at Infinity and Horizontal Asymptotes	**LIM-2:** Reasoning with definitions, theorems, and properties can be used to justify claims about continuity. **LIM-2.D:** Interpret the behavior of functions using limits involving infinity.	**LIM-2.D.3:** The concept of a limit can be extended to include limits at infinity.	Section 1.6, pp. 108-114
		LIM-2.D.4: Limits at infinity describe end behavior.	Section 1.6, pp. 108-114
		LIM-2.D.5: Relative magnitudes of functions and their rates of change can be compared using limits.	Section 1.6, pp. 108-114

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 1.16: Working with the Intermediate Value Theorem (IVT)	**FUN-1:** Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior. **FUN-1.A:** Explain the behavior of a function on an interval using the Intermediate Value Theorem.	**FUN-1.A .1:** If f is a continuous function on the closed interval $[a, b]$ and d is a number between $f(a)$ and $f(b)$, then the Intermediate Value Theorem guarantees that there is at least one number c between a and b such that $f(c) = d$.	Section 1.4, pp. 94-95

Course: AP® Calculus AB and BC

Unit 2: Differentiation: Definition and Fundamental Properties

Suggested Length: AB ~13-14 class periods
BC ~9-10 class periods

AP® Exam Weighting: AB 10-12%
BC 4-7%

Big Ideas: Change (CHA); Limits (LIM); Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 2.1: Defining Average and Instantaneous Rates of Change at a Point	**CHA-2:** Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. **CHA-2.A:** Determine average rates of change using difference quotients.	**CHA-2.A.1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.	Section 2.1, p. 125 Section 2.2, pp. 142-143
	CHA-2: Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. **CHA-2.B:** Represent the derivative of a function as the limit of a difference quotient.	**CHA-2.B.1:** The instantaneous rate of change of a function at $x = a$ can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted $f'(a)$.	Section 2.1, p. 127 Section 2.2, pp. 142-143
Topic 2.2: Defining the Derivative of a Function and Using Derivative Notation	**CHA-2:** Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.	**CHA-2.B.2:** The derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, provided this limit exists.	Section 2.1, p. 127

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	CHA-2.B: Represent the derivative of a function as the limit of a difference quotient.	**CHA-2.B.3:** For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.	Section 2.1, p. 127
		CHA-2.B.4: The derivative can be represented graphically, numerically, analytically, and verbally.	Section 2.1, pp. 127-128
	CHA-2: Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. **CHA-2.C:** Determine the equation of a line tangent to a curve at a given point.	**CHA-2.C.1:** The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.	Section 2.1, pp. 124-128
Topic 2.3: Estimating Derivatives of a Function at a Point	**CHA-2:** Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. **CHA-2.D:** Estimate derivatives.	**CHA-2.D.1:** The derivative at a point can be estimated from information given in tables or graphs.	Section 2.1, p. 132, Exercises 1-2 AP Exam Practice Questions for Chapter 2, p. 209, Exercise 8
		CHA-2.D.2: Technology can be used to calculate or estimate the value of a derivative of a function at a point.	Section 2.1, p. 130
Topic 2.4: Connecting Differentiability and Continuity: Determining When	**FUN-2:** Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both. **FUN-2.A:**	**FUN-2.A.1:** If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'.	Section 2.1, pp. 129-130

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Derivatives Do and Do Not Exist	Explain the relationship between differentiability and continuity.	**FUN-2.A.2:** A continuous function may fail to be differentiable at a point in its domain.	Section 2.1, pp. 129-130
Topic 2.5: Applying the Power Rule	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.A:** Calculate derivatives of familiar functions.	**FUN-3.A.1:** Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form $f(x) = x^r$.	Section 2.2, pp. 136-137
Topic 2.6: Derivative Rules: Constant, Sum, Difference, and Constant Multiple	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.A:** Calculate derivatives of familiar functions.	**FUN-3.A.2:** Sums, differences, and constant multiples of functions can be differentiated using derivative rules.	Section 2.2, pp. 135, 138-139
		FUN-3.A.3: The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.	Section 2.2, p. 139
Topic 2.7: Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.A:** Calculate derivatives of familiar functions.	**FUN-3.A.4:** Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.	Section 2.2, pp. 140-141 Section 2.4, pp. 165-168
	LIM-3: Reasoning with definitions, theorems, and properties can be used to determine a limit. **LIM-3.A:** Interpret a limit as a definition of a derivative.	**LIM-3.A.1:** In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit.	Section 2.1, p. 127
Topic 2.8: The Product Rule	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation.	**FUN-3.B.1:** Derivatives of products of differentiable functions can be found using the product rule.	Section 2.3, pp. 148-149

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	FUN-3.B: Calculate derivatives of products and quotients of differentiable functions.		
Topic 2.9: The Quotient Rule	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.B:** Calculate derivatives of products and quotients of differentiable functions.	**FUN-3.B.2:** Derivatives of quotients of differentiable functions can be found using the quotient rule.	Section 2.3, pp. 150-152
Topic 2.10: Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.B:** Calculate derivatives of products and quotients of differentiable functions.	**FUN-3.B.3:** Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules.	Section 2.3, pp. 152-153

Course: AP® Calculus AB and BC

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions

Suggested Length: AB ~10-11 class periods
BC ~8-9 class periods

AP® Exam Weighting: AB 9-13%
BC 4-7%

Big Ideas: Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 3.1: The Chain Rule	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.C:** Calculate derivatives of compositions of differentiable functions.	**FUN-3.C.1:** The chain rule provides a way to differentiate composite functions.	Section 2.4, pp. 159-168
Topic 3.2: Implicit Differentiation	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.D:** Calculate derivatives of implicitly defined functions.	**FUN-3.D.1:** The chain rule is the basis for implicit differentiation.	Section 2.5, pp. 174-179
Topic 3.3: Differentiating Inverse Functions	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.E:** Calculate derivatives of inverse and inverse trigonometric functions.	**FUN-3.E.1:** The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists.	Section 2.6, pp. 183-186
Topic 3.4:	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation.	**FUN-3.E.2:** The chain rule applied with the definition of an inverse function, or the formula for the derivative	Section 2.6, pp. 184-186

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Differentiating Inverse Trigonometric Functions	**FUN-3.E:** Calculate derivatives of inverse and inverse trigonometric functions.	of an inverse function, can be used to find the derivatives of inverse trigonometric functions.	
Topic 3.5: Selecting Procedures for Calculating Derivatives			Section 2.4, p. 168 Section 2.6, p. 186
Topic 3.6: Calculating Higher-Order Derivatives	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.F:** Determine higher-order derivatives of a function.	**FUN-3.F.1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher-order derivatives of f.	Section 2.3, p. 153
		FUN-3.F.2: Higher-order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher-order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.	Section 2.3, p. 153

Course: AP® Calculus AB and BC

Unit 4: Contextual Applications of Differentiation

Suggested Length: AB ~10-11 class periods; BC ~6-7 class periods

AP® Exam Weighting: AB 10-15%; BC 6-9%

Big Ideas: Change (CHA), Limits (LIM)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 4.1: Interpreting the Meaning of the Derivative in Context	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.A:** Interpret the meaning of a derivative in context.	**CHA-3.A.1:** The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.	Section 2.1, p. 127
		CHA-3.A.2: The derivative can be used to express information about rates of change in applied contexts.	Section 2.2, pp. 142-143
		CHA-3.A.3: The unit for $f'(x)$ is the unit for f divided by the unit for x.	Section 2.2, pp. 142-143
Topic 4.2: Straight-Line Motion: Connecting Position, Velocity, and Acceleration	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.B:** Calculate rates of change in applied contexts.	**CHA-3.B.1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.	Section 2.2, pp. 142-143; Section 2.3, p. 154; Section 2.7, pp. 193-194
Topic 4.3: Rates of Change in Applied Contexts Other Than Motion	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.C:** Interpret rates of change in applied contexts.	**CHA-3.C.1:** The derivative can be used to solve problems involving rates of change in applied contexts.	Section 2.7, pp. 191-192

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 4.4: Introduction to Related Rates	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.D:** Calculate related rates in applied contexts.	**CHA-3.D.1:** The chain rule is the basis for differentiating variables in a related rates problem with respect to the same independent variable.	Section 2.7, pp. 190-194
		CHA-3.D.2: Other differentiation rules, such as the product rule and the quotient rule, may also be necessary to differentiate all variables with respect to the same independent variable.	Section 2.7, pp. 190-194
Topic 4.5: Solving Related Rates Problems	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.E:** Interpret related rates in applied contexts.	**CHA-3.E.1:** The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.	Section 2.7, pp. 190-194
Topic 4.6: Approximating Values of a Function Using Local Linearity and Linearization	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.F:** Approximate a value on a curve using the equation of a tangent line.	**CHA-3.F.1:** The tangent line is the graph of a locally linear approximation of the function near the point of tangency.	Section 3.7, pp. 267-271
		CHA-3.F.2: For a tangent line approximation, the function's behavior near the point of tangency may determine whether a tangent line value is an underestimate or an overestimate of the corresponding function value.	Section 3.7, pp. 267-271
Topic 4.7: Using L'Hospital's Rule for Determining Limits of Indeterminate Forms	**LIM-4:** L'Hospital's rule allows us to determine the limits of some indeterminate forms. **LIM-4.A:** Determine limits of functions that result in indeterminate forms.	**LIM-4.A.1:** When the ratio of two functions tends to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in the limit, such forms are said to be indeterminate.	Section 7.7, p. 506
		LIM-4.A.2: Limits of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's rule.	Section 7.7, pp. 507-509

Course: AP® Calculus AB and BC

Unit 5: Analytical Applications of Differentiation

Suggested Length: AB ~15-16 class periods
BC ~10-11 class periods

AP® Exam Weighting: AB 15-18%
BC 8-11%

Big Ideas: Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 5.1: Using the Mean Value Theorem	**FUN-1:** Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior. **FUN-1.B:** Justify conclusions about functions by applying the Mean Value Theorem over an interval.	**FUN-1.B.1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), then the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.	Section 3.2, pp. 222-223
Topic 5.2: Extreme Value Theorem, Global Versus Local Extrema, and Critical Points	**FUN-1:** Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior. **FUN-1.C:** Justify conclusions about functions by applying the Extreme Value Theorem.	**FUN-1.C.1:** If a function f is continuous over the interval (a,b), then the Extreme Value Theorem guarantees that f has at least one minimum value and at least one maximum value on (a,b).	Section 3.1, p. 212
Topic 5.3: Determining Intervals on Which a Function Is Increasing or Decreasing	**FUN-4:** A function's derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.1:** The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing.	Section 3.3, pp. 227-232
Topic 5.4: Using the First Derivative Test to	**FUN-4:** A function's derivative can be used to understand some behaviors of the function.	**FUN-4.A.2:** The first derivative of a function can determine	Section 3.1, pp. 212-216

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Determine Relative (Local) Extrema	**FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	the location of relative (local) extrema of the function.	
Topic 5.5: Using the Candidates Test to Determine Absolute (Global) Extrema	**FUN-4:** A function's derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.3:** Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints.	Section 3.1, pp. 212-216
Topic 5.6: Determining Concavity of Functions over Their Domains	**FUN-4:** A function's derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.4:** The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval.	Section 3.4, p. 237
		FUN-4.A.5: The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity.	Section 3.4, pp. 238-241
		FUN-4.A.6: The second derivative of a function may be used to locate points of inflection for the graph of the original function.	Section 3.4, pp. 239-240
Topic 5.7: Using the Second Derivative Test to Find Extrema	**FUN-4:** A function's derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.7:** The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum.	Section 3.4, p. 241
		FUN-4.A.8: When a continuous function has only one critical point on an interval on its domain and the critical point corresponds to a relative (local) extremum of the function on the interval, then that critical point also corresponds to the absolute (global) extremum of the function on the interval.	Section 3.4, p. 241

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 5.8: Sketching Graphs of Functions and Their Derivatives	**FUN-4:** A function’s derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.9:** Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.	Section 3.5, pp. 245-252
		FUN-4.A.10: Graphical, numerical, and analytical information from f' and f'' can be used to predict and explain the behavior of f .	Section 3.5, pp. 245-252
Topic 5.9: Connecting a Function, Its First Derivative, and Its Second Derivative	**FUN-4:** A function’s derivative can be used to understand some behaviors of the function. **FUN-4.A:** Justify conclusions about the behavior of a function based on the behavior of its derivatives.	**FUN-4.A.11:** Key features of the graphs of f, f', and f'' are related to one another.	Section 3.5, pp. 245-252
Topic 5.10: Introduction to Optimization Problems	**FUN-4:** A function’s derivative can be used to understand some behaviors of the function. **FUN-4.B:** Calculate minimum and maximum values in applied contexts or analysis of functions.	**FUN-4.B.1:** The derivative can be used to solve optimization problems; that is, finding a minimum or maximum value of a function on a given interval.	Section 3.6, pp. 257-261
Topic 5.11: Solving Optimization Problems	**FUN-4:** A function’s derivative can be used to understand some behaviors of the function. **FUN-4.C:** Interpret minimum and maximum values calculated in applied contexts.	**FUN-4.C.1:** Minimum and maximum values of a function take on specific meanings in applied contexts.	Section 3.6, pp. 257-261
Topic 5.12: Exploring Behaviors of Implicit Relations	**FUN-4:** A function’s derivative can be used to understand some behaviors of the function. **FUN-4.D:** Determine critical points of implicit relations.	**FUN-4.D.1:** A point on an implicit relation where the first derivative equals zero or does not exist is a critical point of the function.	Section 3.1, pp. 213-216

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	FUN-4: A function's derivative can be used to understand some behaviors of the function. **FUN-4.E:** Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.	**FUN-4.E.1:** Applications of derivatives can be extended to implicitly defined functions.	Section 2.5, pp. 174-179
		FUN-4.E.2: Second derivatives involving implicit differentiation may be relations of $x, y, \text{and } \frac{dy}{dx}$.	Section 2.5, p. 178

Course: AP® Calculus AB and BC

Unit 6: Integration and Accumulation of Change

Suggested Length: AB ~18-20 class periods
BC ~15-16 class periods

AP® Exam Weighting: AB 17-20%
BC 17-20%

Big Ideas: Change (CHA); Limits (LIM); Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 6.1: Exploring Accumulations of Change	**CHA-4:** Definite integrals allow us to solve problems involving the accumulation of change over an interval. **CHA-4.A:** Interpret the meaning of areas associated with the graph of a rate of change in context.	**CHA-4.A.1:** The area of the region between the graph of a rate of change function and the *x*-axis gives the accumulation of change.	Section 4.4, p. 323 Section 4.5, p. 330
		CHA-4.A.2: In some cases, accumulation of change can be evaluated by using geometry.	Section 4.2, pp. 292-293
		CHA-4.A.3: If a rate of change is positive (negative) over an interval, then the accumulated change is positive (negative).	Section 4.5, p. 331
		CHA-4.A.4: The unit for the area of a region defined by rate of change is the unit for the rate of change multiplied by the unit for the independent variable.	Section 4.5, p. 330
Topic 6.2: Approximating Areas with Riemann Sums	**LIM-5:** Definite integrals can be approximated using geometric and numerical methods. **LIM-5.A:** Approximate a definite integral using geometric and numerical methods.	**LIM-5.A.1:** Definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally.	Section 4.3, pp. 304-311
		LIM-5.A.2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint	Section 4.3, pp. 302-305, 309-311

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.	
		LIM-5.A.3: Definite integrals can be approximated using numerical methods, with or without technology.	Section 4.3, pp. 309-310
		LIM-5.A.4: Depending on the behavior of a function, it may be possible to determine whether an approximation for a definite integral is an underestimate or overestimate for the value of the definite integral.	Section 4.3, pp. 309-310
Topic 6.3: Riemann Sums, Summation Notation, and Definite Integral Notation	**LIM-5:** Definite integrals can be approximated using geometric and numerical methods. **LIM-5.B:** Interpret the limiting case of the Riemann sum as a definite integral.	**LIM-5.B.1:** The limit of an approximating Riemann sum can be interpreted as a definite integral.	Section 4.3, pp. 304-305
		LIM-5.B.2: A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.	Section 4.3, pp. 302-303
	LIM-5: Definite integrals can be approximated using geometric and numerical methods. **LIM-5.C:** Represent the limiting case of the Riemann sum as a definite integral.	**LIM-5.C.1:** The definite integral of a continuous function f over the interval $[a,b]$, denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where n is the number of subintervals, Δx_i is the width of the ith subinterval, and x_i^* is a value in the ith subinterval.	Section 4.3, p. 304

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		LIM-5.C.2: A definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.	Section 4.3, p. 304
Topic 6.4: The Fundamental Theorem of Calculus and Accumulation Functions	**FUN-5:** The Fundamental Theorem of Calculus connects differentiation and integration. **FUN-5.A:** Represent accumulation functions using definite integrals.	**FUN-5.A.1:** The definite integral can be used to define new functions.	Section 4.4, pp. 323-325
		FUN-5.A.2: If f is a continuous function on an interval containing a, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, where x is in the interval.	Section 4.4, pp. 324-325
Topic 6.5: Interpreting the Behavior of Accumulation Functions Involving Area	**FUN-5:** The Fundamental Theorem of Calculus connects differentiation and integration. **FUN-5.A:** Represent accumulation functions using definite integrals.	**FUN-5.A.3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$.	Section 4.4, pp. 323-325
Topic 6.6: Applying Properties of Definite Integrals	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.A:** Calculate a definite integral using areas and properties of definite integrals.	**FUN-6.A.1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.	Section 4.4, pp. 320-321
		FUN-6.A.2: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.	Section 4.3, pp. 307-308
		FUN-6.A.3: The definition of the definite integral may be extended to functions with removable or jump discontinuities.	Section 4.3, p. 308

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 6.7: The Fundamental Theorem of Calculus and Definite Integrals	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.B:** Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.	**FUN-6.B.1:** An antiderivative of a function f is a function g whose derivative is f.	Section 4.1, pp. 280-281
		FUN-6.B.2: If a function f is continuous on an interval containing a, the function defined by $F(x) = \int_a^x f(t)dt$ is an antiderivative of f for x in the interval.	Section 4.4, p. 324-325
		FUN-6.B.3: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)dx = F(b) - F(a)$.	Section 4.4, p. 317-319
Topic 6.8: Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.C:** Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.	**FUN-6.C.1:** $\int f(x)dx$ is an indefinite integral of the function f and can be expressed as $\int f(x)dx = F(x) + C$, where $F'(x) = f(x)$ and C is any constant.	Section 4.1, pp. 280-281
		FUN-6.C.2: Differentiation rules provide the foundation for finding antiderivatives.	Section 4.1, pp. 282-284
		FUN-6.C.3: Many functions do not have closed-form antiderivatives.	Section 4.3, p. 309
Topic 6.9: Integrating Using Substitution	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.	**FUN-6.D.1:** Substitution of variables is a technique for finding antiderivatives.	Section 4.6, pp. 337-342 Section 7.1, pp. 456-459

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	FUN-6.D: For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals.	**FUN-6.D.2:** For a definite integral, substitution of variables requires corresponding changes to the limits of integration.	Section 4.6, pp. 340-342
Topic 6.10: Integrating Functions Using Long Division and Completing the Square	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.D:** For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals.	**FUN-6.D.3:** Techniques for finding antiderivatives include rearrangements into equivalent forms, such as long division and completing the square.	Section 4.7, p. 349 Section 4.8, p. 358 Section 7.1, pp. 456, 459
Topic 6.11: Integrating Using Integration by Parts **BC ONLY**	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.E:** For integrands requiring integration by parts: (a) Determine indefinite integrals. **BC ONLY** (b) Evaluate definite integrals. **BC ONLY**	**FUN-6.E.1:** Integration by parts is a technique for finding antiderivatives. **BC ONLY**	Section 7.2, pp. 463-468
Topic 6.12: Integrating Using Linear Partial Fractions **BC ONLY**	**FUN-6:** Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. **FUN-6.F:** For integrands requiring integration by linear partial fractions:	**FUN-6.F.1:** Some rational functions can be decomposed into sums of ratios of linear, nonrepeating factors to which basic integration techniques can be applied. **BC ONLY**	Section 7.5, pp. 491-497

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	(a) Determine indefinite integrals. **BC ONLY** (b) Evaluate definite integrals. **BC ONLY**		
Topic 6.13: Evaluating Improper Integrals **BC ONLY**	**LIM-6** The use of limits allows us to show that the areas of unbounded regions may be finite. **LIM-6.A:** Evaluate an improper integral or determine that the integral diverges. **BC ONLY**	**LIM-6.A.1:** An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration. **BC ONLY**	Section 7.8, p. 517
		LIM-6.A.2: Improper integrals can be determined using limits of definite integrals. **BC ONLY**	Section 7.8, pp. 517-523
Topic 6.14: Selecting Techniques for Antidifferentiation			Section 4.1, p. 282 Section 4.8, p. 359 Section 7.1, p. 459 Sections 7.2-7.6, pp. 463-505

Course: AP® Calculus AB and BC

Unit 7: Differential Equations

Suggested Length: AB ~8-9 class periods
BC ~9-10 class periods

AP® Exam Weighting: AB 6-12%
BC 6-9%

Big Ideas: Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 7.1: Modeling Situations with Differential Equations	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.A:** Interpret verbal statements of problems as differential equations involving a derivative expression.	**FUN-7.A.1:** Differential equations relate a function of an independent variable and the function's derivatives.	Section 4.1, p. 281 Section 5.1, p. 370
Topic 7.2: Verifying Solutions for Differential Equations	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.B:** Verify solutions to differential equations.	**FUN-7.B.1:** Derivatives can be used to verify that a function is a solution to a given differential equation.	Section 5.1, pp. 370-371
		FUN-7.B.2: There may be infinitely many general solutions to a differential equation.	Section 5.1, pp. 370-371
Topic 7.3: Sketching Slope Fields	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.C:** Estimate solutions to differential equations.	**FUN-7.C.1:** A slope field is a graphical representation of a differential equation on a finite set of points in the plane.	Section 5.1, pp. 372-373
		FUN-7.C.2: Slope fields provide information about the behavior of solutions to first-order differential equations.	Section 5.1, pp. 372-373

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 7.4: Reasoning Using Slope Fields	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.C:** Estimate solutions to differential equations.	**FUN-7.C.3:** Solutions to differential equations are functions or families of functions.	Section 5.1, pp. 370-371
Topic 7.5: Approximating Solutions Using Euler's Method **BC ONLY**	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.C:** Estimate solutions to differential equations.	**FUN-7.C.4:** Euler's method provides a procedure for approximating a solution to a differential equation or a point on a solution curve. **BC ONLY**	Section 5.1, p. 374
Topic 7.6: Finding General Solutions Using Separation of Variables	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.D:** Determine general solutions to differential equations.	**FUN-7.D.1:** Some differential equations can be solved by separation of variables.	Section 5.2, p. 379-380, 383 Section 5.3, pp. 387-392
		FUN-7.D.2: Antidifferentiation can be used to find general solutions to differential equations.	Section 4.1, p.281 Section 5.2, p. 379-380, 383 Section 5.3, p. 387
Topic 7.7: Finding Particular Solutions Using Initial Conditions and Separation of Variables	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.E:** Determine particular solutions to differential equations.	**FUN-7.E.1:** A general solution may describe infinitely many solutions to a differential equation. There is only one particular solution passing through a given point.	Section 4.1, p. 285 Section 5.1, pp. 370-371 Section 5.2, p. 379-380, 383 Section 5.3, pp. 387-388
		FUN-7.E.2: The function F defined by $F(x) = y_0 + \int_a^x f(t)\,dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$, satisfying $F(a) = y_0$.	Section 4.1, pp. 285-286 Section 5.3, p. 388

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		FUN-7.E.3: Solutions to differential equations may be subject to domain restrictions.	Section 5.3, p. 388, Example 3
Topic 7.8: Exponential Models with Differential Equations	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.F:** Interpret the meaning of a differential equation and its variables in context.	**FUN-7.F.1:** Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay.	Section 5.2, pp. 380-383
		FUN-7.F.2: The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.	Section 5.2, p. 380
	FUN-7: Solving differential equations allows us to determine functions and develop models. **FUN-7.G:** Determine general and particular solutions for problems involving differential equations in context.	**FUN-7.G.1:** The exponential growth and decay model, $\frac{dy}{dt} = ky$, with initial condition $y = y_0$ when $t = 0$, has solutions of the form $y = y_0 e^{kt}$.	Section 5.2, pp. 380-383
Topic 7.9: Logistic Models with Differential Equations **BC ONLY**	**FUN-7:** Solving differential equations allows us to determine functions and develop models. **FUN-7.H:** Interpret the meaning of the logistic growth model in context. **BC ONLY**	**FUN-7.H.1:** The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a - y)$. **BC ONLY**	Section 5.4, pp. 397-401
		FUN-7.H.2: The logistic differential equation and initial conditions can be interpreted without solving the differential equation. **BC ONLY**	Section 5.4, pp. 398, 400

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		FUN-7.H.3: The limiting value (carrying capacity) of a logistic differential equation as the independent variable approaches infinity can be determined using the logistic growth model and initial conditions. **BC ONLY**	Section 5.4, p. 401
		FUN-7.H.4: The value of the dependent variable in a logistic differential equation at the point when it is changing fastest can be determined using the logistic growth model and initial conditions. **BC ONLY**	Section 5.4, p. 403, Exercise 31

Course: AP® Calculus AB and BC

Unit 8: Applications of Integration

Suggested Length: AB ~19-20 class periods
BC ~13-14 class periods

AP® Exam Weighting: AB 10-15%
BC 6-9%

Big Ideas: Change (CHA)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 8.1: Finding the Average Value of a Function on an Interval	**CHA-4:** Definite integrals allow us to solve problems involving the accumulation of change over an interval. **CHA-4.B:** Determine the average value of a function using definite integrals.	**CHA-4.B.1:** The average value of a continuous function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.	Section 4.4, pp. 321-322
Topic 8.2: Connecting Position, Velocity, and Acceleration of Functions Using Integrals	**CHA-4:** Definite integrals allow us to solve problems involving the accumulation of change over an interval. **CHA-4.C:** Determine values for positions and rates of change using definite integrals in problems involving rectilinear motion.	**CHA-4.C.1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.	Section 4.5, pp. 329-330
Topic 8.3: Using Accumulation Functions and Definite Integrals in Applied Contexts	**CHA-4:** Definite integrals allow us to solve problems involving the accumulation of change over an interval. **CHA-4.D:** Interpret the meaning of a definite integral in accumulation problems.	**CHA-4.D.1:** A function defined as an integral represents an accumulation of a rate of change.	Section 4.4, pp. 323-324
		CHA-4.D.2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.	Section 4.5, pp. 329-330

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	CHA-4: Definite integrals allow us to solve problems involving the accumulation of change over an interval. **CHA-4.E:** Determine net change using definite integrals in applied contexts.	**CHA-4.E.1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.	Section 4.5, pp. 329-331
Topic 8.4: Finding the Area Between Curves Expressed as Functions of *x*	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.A:** Calculate areas in the plane using the definite integral.	**CHA-5.A.1:** Areas of regions in the plane can be calculated with definite integrals.	Section 6.1, pp. 410-415
Topic 8.5: Finding the Area Between Curves Expressed as Functions of *y*	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.A:** Calculate areas in the plane using the definite integral.	**CHA-5.A.2:** Areas of regions in the plane can be calculated using functions of either *x* or *y*.	Section 6.1, p. 414
Topic 8.6: Finding the Area Between Curves That Intersect at More Than Two Points	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.A:** Calculate areas in the plane using the definite integral.	**CHA-5.A.3:** Areas of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.	Section 6.1, p. 413, 414
Topic 8.7: Volumes with Cross Sections: Squares and Rectangles	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.	**CHA-5.B.1:** Volumes of solids with square and rectangular cross sections can be found using definite integrals and the area formulas for these shapes.	Section 6.2, pp. 425-426

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	CHA-5.B: Calculate volumes of solids with known cross sections using definite integrals.		
Topic 8.8: Volumes with Cross Sections: Triangles and Semicircles	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.B:** Calculate volumes of solids with known cross sections using definite integrals.	**CHA-5.B.2:** Volumes of solids with triangular cross sections can be found using definite integrals and the area formulas for these shapes.	Section 6.2, pp. 425-426
		CHA-5.B.3: Volumes of solids with semicircular and other geometrically defined cross sections can be found using definite integrals and the area formulas for these shapes.	Section 6.2, pp. 425-426
Topic 8.9: Volume with Disc Method: Revolving Around the *x*- or *y*-Axis	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.C:** Calculate volumes of solids of revolution using definite integrals.	**CHA-5.C.1:** Volumes of solids of revolution around the *x*- or *y*-axis may be found by using definite integrals with the disc method.	Section 6.2, pp. 420-422
Topic 8.10: Volume with Disc Method: Revolving Around Other Axes	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.C:** Calculate volumes of solids of revolution using definite integrals.	**CHA-5.C.2:** Volumes of solids of revolution around any horizontal or vertical line in the plane may be found by using definite integrals with the disc method.	Section 6.2, pp. 420-422

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 8.11: Volume with Washer Method: Revolving Around the *x*- or *y*-Axis	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.C:** Calculate volumes of solids of revolution using definite integrals.	**CHA-5.C.3:** Volumes of solids of revolution around the *x*- or *y*-axis whose cross-sections are ring shaped may be found using definite integrals with the washer method.	Section 6.2, pp. 423-425
Topic 8.12: Volume with Washer Method: Revolving Around Other Axes	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.C:** Calculate volumes of solids of revolution using definite integrals.	**CHA-5.C.4:** Volumes of solids of revolution around any horizontal or vertical line whose cross sections are ring shaped may be found using definite integrals with the washer method.	Section 6.2, pp. 423-425
Topic 8.13: The Arc Length of a Smooth, Planar Curve and Distance Traveled **BC ONLY**	**CHA-6:** Definite integrals allow us to solve problems involving the accumulation of change in length over an interval. **CHA-6.A:** Determine the length of a curve in the plane defined by a function, using a definite integral. **BC ONLY**	**CHA-6.A.1:** The length of a planar curve defined by a function can be calculated using a definite integral. **BC ONLY**	Section 6.4, pp. 440-443

Course: AP® Calculus BC Only

Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions

Suggested Length: AB Not Applicable
BC ~10-11 class periods

AP® Exam Weighting: AB Not Applicable
BC 11-12%

Big Ideas: Change (CHA); Analysis of Functions (FUN)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 9.1: Defining and Differentiating Parametric Equations (**BC ONLY**)	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.G:** Calculate derivatives of parametric functions. **BC ONLY**	**CHA-3.G.1:** Methods for calculating derivatives of real-valued functions can be extended to parametric functions. **BC ONLY**	Section 9.3, pp. 655-658
		CHA-3.G.2: For a curve defined parametrically, the value of $\frac{dy}{dx}$ at a point on the curve is the slope of the line tangent to the curve at that point. $\frac{dy}{dx}$, the slope of the line tangent to a curve defined using parametric equations, can be determined by dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$, provided $\frac{dx}{dt}$ does not equal zero. **BC ONLY**	Section 9.3, pp. 655-658
Topic 9.2: Second Derivatives of Parametric Equations **BC ONLY**	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.G:** Calculate derivatives of parametric functions. **BC ONLY**	**CHA-3.G.3:** $\frac{d^2y}{dx^2}$ can be calculated by dividing $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ by $\frac{dx}{dt}$. **BC ONLY**	Section 9.3, p. 656

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 9.3: Finding Arc Lengths of Curves Given by Parametric Equations **BC ONLY**	**CHA-6:** Definite integrals allow us to solve problems involving the accumulation of change in length over an interval. **CHA-6.B:** Determine the length of a curve in the plane defined by parametric functions, using a definite integral. **BC ONLY**	**CHA-6.B.1:** The length of a parametrically defined curve can be calculated using a definite integral. **BC ONLY**	Section 9.3, pp. 657-658
Topic 9.4: Defining and Differentiating Vector-Valued Functions **BC ONLY**	**CHA-3:** Derivatives allow us to solve real-world problems involving rates of change. **CHA-3.H:** Calculate derivatives of vector-valued functions. **BC ONLY**	**CHA-3.H.1:** Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions. **BC ONLY**	Section 9.7, pp. 689-693
Topic 9.5: Integrating Vector-Valued Functions **BC ONLY**	**FUN-8:** Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane. **FUN-8.A:** Determine a particular solution given a rate vector and initial conditions. **BC ONLY**	**FUN-8.A.1** Methods for calculating integrals of real-valued functions can be extended to parametric or vector-valued functions. **BC ONLY**	Section 9.7, p. 694
Topic 9.6: Solving Motion Problems Using Parametric and Vector-Valued Functions **BC ONLY**	**FUN-8:** Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane. **FUN-8.B:** Determine values for positions and rates of change in problems involving planar motion. **BC ONLY**	**FUN-8.B.1:** Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions. **BC ONLY**	Section 9.8, pp. 698-700
		FUN-8.B.2: For a particle in planar motion over an interval of time, the definite integral of the velocity vector represents the particle's displacement (net change in position) over the interval of time, from which we might determine its position. The definite integral of speed represents the particle's	Section 9.8, p. 701

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		total distance traveled over the interval of time. **BC ONLY**	
Topic 9.7: Defining Polar Coordinates and Differentiating in Polar Form **BC ONLY**	**FUN-3:** Recognizing opportunities to apply derivative rules can simplify differentiation. **FUN-3.G:** Calculate derivatives of functions written in polar coordinates. **BC ONLY**	**FUN-3.G.1:** Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates. **BC ONLY**	Section 9.4, pp. 663-669
		FUN-3.G.2: For a curve given by a polar equation $r = f(\theta)$, derivatives of r, x, and y with respect to θ, and first and second derivatives of y with respect to x can provide information about the curve. **BC ONLY**	Section 9.4, pp. 667-668
Topic 9.8: Finding the Area of a Polar Region or the Area Bounded by a Single Polar Curve **BC ONLY**	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.D:** Calculate areas of regions defined by polar curves using definite integrals. **BC ONLY**	**CHA-5.D.1:** The concept of calculating areas in rectangular coordinates can be extended to polar coordinates. **BC ONLY**	Section 9.5, pp. 673-674
Topic 9.9: Finding the Area of the Region Bounded by Two Polar Curves **BC ONLY**	**CHA-5:** Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. **CHA-5.D:** Calculate areas of regions defined by polar curves using definite integrals. **BC ONLY**	**CHA-5.D.2:** Areas of regions bounded by polar curves can be calculated with definite integrals. **BC ONLY**	Section 9.5, p. 676

Course: AP® Calculus BC Only

Unit 10: Infinite Sequences and Series

Suggested Length: AB Not Applicable
BC ~17-18 class periods

AP® Exam Weighting: AB Not Applicable
BC 17-18%

Big Ideas: Limits (LIM)

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 10.1: Defining Convergent and Divergent Infinite Series **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.1:** The *n*th partial sum is defined as the sum of the first *n* terms of a series. **BC ONLY**	Section 8.2, pp. 545-547
		LIM-7.A.2: An infinite series of numbers converges to a real number *S* (or has sum *S*), if and only if the limit of its sequence of partial sums exists and equals *S*. **BC ONLY**	Section 8.2, pp. 545-547
Topic 10.2: Working with Geometric Series **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.3:** A geometric series is a series with a constant ratio between successive terms. **BC ONLY**	Section 8.2, pp. 547-548
		LIM-7.A.4: If *a* is a real number and *r* is a real number such that $\lvert r\rvert < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$. **BC ONLY**	Section 8.2, pp. 547-548
Topic 10.3: The *n*th-Term Test for Divergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.5:** The *n*th-term test ~~is~~ a test for divergence of a series. **BC ONLY**	Section 8.2, pp. 549-550

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 10.4: Integral Test for Convergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.6:** The integral test is a method to determine whether a series converges or diverges. **BC ONLY**	Section 8.3, pp. 555-556
Topic 10.5: Harmonic Series and *p*-Series **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.7:** In addition to geometric series, common series of numbers include the harmonic series, the alternating harmonic series, and *p*-series. **BC ONLY**	Section 8.3, pp. 557-558
Topic 10.6: Comparison Tests for Convergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.8:** The comparison test is a method to determine whether a series converges or diverges. **BC ONLY**	Section 8.4, pp. 562-563
		LIM-7.A.9: The limit comparison test is a method to determine whether a series converges or diverges. **BC ONLY**	Section 8.4, pp. 564-565
Topic 10.7: Alternating Series Test for Convergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.10:** The alternating series test is a method to determine whether an alternating series converges. **BC ONLY**	Section 8.5, pp. 569-574
Topic 10.8: Ratio Test for Convergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.11:** The ratio test is a method to determine whether a series of numbers converges or diverges. **BC ONLY**	Section 8.6, pp. 577-579

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
Topic 10.9: Determining Absolute or Conditional Convergence **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.A:** Determine whether a series converges or diverges. **BC ONLY**	**LIM-7.A.12:** A series may be absolutely convergent, conditionally convergent, or divergent. **BC ONLY**	Section 8.5, pp. 572-573
		LIM-7.A.13: If a series converges absolutely, then it converges. **BC ONLY**	Section 8.5, pp. 572-573
		LIM-7.A.14: If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value. **BC ONLY**	Section 8.5, p. 574
Topic 10.10: Alternating Series Error Bound **BC ONLY**	**LIM-7:** Applying limits may allow us to determine the finite sum of infinitely many terms. **LIM-7.B:** Approximate the sum of a series. **BC ONLY**	**LIM-7.B.1:** If an alternating series converges by the alternating series test, then the alternating series error bound can be used to bound how far a partial sum is from the value of the infinite series. **BC ONLY**	Section 8.5, p. 571
Topic 10.11: Finding Taylor Polynomial Approximations of Functions **BC ONLY**	**LIM-8:** Power series allow us to represent associated functions on an appropriate interval. **LIM-8.A:** Represent a function at a point as a Taylor polynomial. **BC ONLY**	**LIM-8.A.1:** The coefficient of the *n*th-degree term in a Taylor polynomial for a function *f* centered at $x = a$ is $\frac{f^{(n)}(a)}{n!}$. **BC ONLY**	Section 8.7, pp. 588-593
		LIM-8.A.2: In many cases, as the degree of a Taylor polynomial increases, the *n*th-degree polynomial will approach the original function over some interval. **BC ONLY**	Section 8.7, pp. 588-590
	LIM-8: Power series allow us to represent associated functions on an appropriate interval. **LIM-8.B:**	**LIM-8.B.1:** Taylor polynomials for a function f centered at $x = a$ can be used to approximate function values of f near $x = a$. **BC ONLY**	Section 8.7, pp. 591

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
	Approximate function values using a Taylor polynomial. **BC ONLY**		
Topic 10.12: Lagrange Error Bound **BC ONLY**	**LIM-8:** Power series allow us to represent associated functions on an appropriate interval. **LIM-8.C:** Determine the error bound associated with a Taylor polynomial approximation. **BC ONLY**	**LIM-8.C.1:** The Lagrange error bound can be used to determine a maximum interval for the error of a Taylor polynomial approximation to a function. **BC ONLY**	Section 8.7, p. 592-593
		LIM-8.C.2: In some situations, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the value of a function. **BC ONLY**	Section 8.9, p. 610 AP Exam Practice Questions for Chapter 8, p. 629, Exercise 9
Topic 10.13: Radius and Interval of Convergence of Power Series (**BC ONLY**)	**LIM-8:** Power series allow us to represent associated functions on an appropriate interval. **LIM-8.D:** Determine the radius of convergence and interval of convergence for a power series. **BC ONLY**	**LIM-8.D.1:** A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x-r)^n$, where *n* is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and *r* is a real number. **BC ONLY**	Section 8.8, p. 597
		LIM-8.D.2: If a power series converges, it either converges at a single point or has an interval of convergence. **BC ONLY**	Section 8.8, pp. 598-599
		LIM-8.D.3: The ratio test can be used to determine the radius of convergence of a power series. **BC ONLY**	Section 8.8, p. 599
		LIM-8.D.4: The radius of convergence of a power series can be used to identify an open interval on which the series converges, but it is necessary to test both endpoints of the interval to determine the interval of convergence. **BC ONLY**	Section 8.8, pp. 600-601

Topic	Enduring Understanding and Learning Objective	Essential Knowledge	Text Section(s)
		LIM-8.D.5: If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval. **BC ONLY**	Section 8.10, pp. 614-615
		LIM-8.D.6: The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series. **BC ONLY**	Section 8.8, pp. 602-603
Topic 10.14: Finding Taylor or Maclaurin Series for a Function **BC ONLY**	**LIM-8:** Power series allow us to represent associated functions on an appropriate interval. **LIM-8.E:** Represent a function as a Taylor series or a Maclaurin series. **BC ONLY**	**LIM-8.E.1:** A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$. **BC ONLY**	Section 8.10, pp. 614-616
	LIM-8: Power series allow us to represent associated functions on an appropriate interval. **LIM-8.F:** Interpret Taylor series and Maclaurin series. **BC ONLY**	**LIM-8.F.1:** The Maclaurin series for $\frac{1}{1-x}$ is a geometric series. **BC ONLY**	Section 8.9, p. 607
		LIM-8.F.2: The Maclaurin series for sin x, cos x, and e^x provides the foundation for constructing the Maclaurin series for other functions. **BC ONLY**	Section 8.10, pp. 617-622
Topic 10.15: Representing Functions as Power Series **BC ONLY**	**LIM-8:** Power series allow us to represent associated functions on an appropriate interval. **LIM-8.G:** Represent a given function as a power series. **BC ONLY**	**LIM-8.G.1:** Using a known series, a power series for a given function can be derived using operations such as term-by-term differentiation or term-by-term integration, and by various methods (e.g., algebraic processes, substitutions, or using properties of geometric series). **BC ONLY**	Section 8.9, pp. 607-611

